AYAZAĞA IŞIK LİSESİ

“ Önce İyi İnsan Yetiştirir ”

2019-2020 Academic Year

For 2021 May Exam Session Candidates

Mathematics:Analysis and Approches
HL Course Booklet

Accredited by CIS

Contents: **Page Number**

IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who
understand that other people, with their differences, can also be right.

IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world. IB learners strive to be:

Inquirers They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy learning and this love of learning will be sustained throughout their lives.

Knowledgeable They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines.

Thinkers They exercise initiative in applying thinking skills critically and creatively to recognizeand approach complex problems, and make reasoned, ethical decisions.

Communicators They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others.

Principled They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them.

Open-minded They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to growfrom the experience.

Caring They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment.

Risk-takers They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are brave and articulate in defending their beliefs.

Balanced They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others.

Reflective They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.

Mathematics analysis and approaches

This course recognizes the need for analytical expertise in a world where innovation is increasingly dependent on a deep understanding of mathematics. This course includes topics that are both traditionally part of a pre-university mathematics course (for example, functions, trigonometry, calculus) as well as topics that are amenable to investigation, conjecture and proof, for instance the study of sequences and series at both SL and HL, and proof by induction at HL. The course allows the use of technology, as fluency in relevant mathematical software and hand-held technology is important regardless of choice of course. However, Mathematics: analysis and approaches has a strong emphasis on the ability to construct, communicate and justify correct mathematical arguments.

Mathematics: analysis and approaches: Distinction between SL and HL

Students who choose Mathematics: analysis and approaches at SL or HL should be comfortable in the manipulation of algebraic expressions and enjoy the recognition of patterns and understand the mathematical generalization of these patterns. Students who wish to take Mathematics: analysis and approaches at higher level will have strong algebraic skills and the ability to understand simple proof. They will be students who enjoy spending time with problems and get pleasure and satisfaction from solving challenging problems.

Group 5 aims

The aims of all DP mathematics courses are to enable students to:

1. develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
2. develop an understanding of the concepts, principles and nature of mathematics
3. communicate mathematics clearly, concisely and confidently in a variety of contexts
4. develop logical and creative thinking, and patience and persistence in problem solving to instil confidence in using mathematics
5. employ and refine their powers of abstraction and generalization
6. take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities
7. appreciate how developments in technology and mathematics influence each other 8. appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics
9. appreciate the universality of mathematics and its multicultural, international and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course
11. develop the ability to reflect critically upon their own work and the work of others
12. independently and collaboratively extend their understanding of mathematics.

Maths HL Syllabus Outline

Syllabus component	Teaching hours
	HL
All topics are compulsory. Students must study all the sub-topics in each of the topics in the syllabus as listed in this guide. Students are also required to be familiar with the topics listed as prior learning.	
Topic 1 Number and Algebra	39
Topic 2 Functions	32
Topic 3 Geometry and trigonometry	51
Topic 4 Statistics and probability	33
Topic 5 Calculus	55
The toolkit and the mathematical exploration Investigative, problem-solving and modelling skills development leading to an individual exploration. The exploration is a piece of written work that involves investigating an area of mathematics.	30
Total teaching hours	240

Assessment Objectives

Problem-solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP mathematics HL course, students will be expected to demonstrate the following.

1. **Knowledge and understanding**: recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.

2. **Problem-solving:** recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems.

3. **Communication and interpretation**: transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation.

4. **Technology:** use technology, accurately, appropriately and efficiently both to explore new ideas and to solve problems.

5. **Reasoning:** construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions.

6. **Inquiry approaches**: investigate unfamiliar situations, both abstract and real-world, involving organizing and analyzing information, making conjectures, drawing conclusions and testing their validity

Assessment Outline

MATHS HL	Weighting
External assessment (5 hours)	80 %
Paper 1 (2 hours) No calculator allowed. (110 marks) • **Section A** (55 marks) Compulsory short-response questions based on the core syllabus. • **Section B** (55 marks) Compulsory extended-response questions based on the core syllabus.	30 %
Paper 2 (2 hours) Graphic display calculator required. (110 marks) • **Section A** (55 marks) Compulsory short-response questions based on the core syllabus. • **Section B** (55 marks) Compulsory extended-response questions based on the core syllabus.	30 %
Paper 3 (1 hour) Graphic display calculator required. (55 marks) Compulsory extended-response questions based mainly on the syllabus options	20 %
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics HL is an individual exploration. This is a piece of written work that involves investigating an area of mathematics(20 marks)	20 %

Assessment Criteria

External assessment details

Paper 1,2 and paper 3

These papers are externally set and externally marked. Together, they contribute 80% of the final mark for the course. These papers are designed to allow students to demonstrate what they know and what they can do.
Papers 1 and 2 will contain some questions, or parts of questions, which are common with SL.

Calculators

Paper 1

Students are not permitted access to any calculator. Questions will mainly involve analytic approaches to solutions, rather than requiring the use of a GDC. The paper is not intended to require complicated calculations, with the potential for careless errors. However, questions will include some arithmetical manipulations when they are essential to the development of the question.

Paper 2

Students must have access to a GDC at all times. However, not all questions will necessarily require the use of the GDC.

Paper 3

Students must have access to a GDC at all times. However, not all question parts will necessarily require the use of the GDC.

Mathematics HL formula booklet

Each student must have access to a clean copy of the formula booklet during the examination. It is the responsibility of the school to provide and ensure that there are sufficient copies available for all students.

Awarding of marks

Marks are awarded for method, accuracy, answers and reasoning, including interpretation. In papers 1, 2 and 3, full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations (in the form of, for example diagrams, graphs or calculations). Where an answer is incorrect, some marks may be given for correct method, provided that this is shown by written working. All students should therefore be advised to show their working.

Internal assessment Details

Internal assessment enables students to demonstrate the application of their skills and knowledge, and to pursue their personal interests. Internal assessment in Mathematics HL is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. It is marked according to five assessment criteria.

Guidance and authenticity: The exploration submitted for internal assessment must be the student's own work. However, it is not the intention that students should decide upon a title or topic and be left to work on the exploration without any further support from the teacher. The teacher should play an important role during both the planning stage and the period when the student is working on the exploration. It is the responsibility of the teacher to ensure that students are familiar with:

- the requirements of the type of work to be internally assessed
- the IB academic honesty policy available on the OCC
- the assessment criteria—students must understand that the work submitted for assessment must address these criteria effectively.

Teachers and students must discuss the exploration. Students should be encouraged to initiate discussions with the teacher to obtain advice and information, and students must not be penalized for seeking guidance.
All students should be able to understand the basic meaning and significance of concepts that relate to academic honesty, especially authenticity and intellectual property. All student work for assessment must be prepared according to the requirements and must entirely their own.

As part of the learning process, teachers can give advice to students on a **first draft** of the exploration. This advice should be in terms of the way the work could be improved, but this first draft must not be heavily annotated or edited by the teacher. The next version handed to the teacher after the first draft must be the final one.

All work submitted to the IB for moderation or assessment must be authenticated by a teacher, and must not include any known instances of suspected or confirmed malpractice. Each student must sign the coversheet for internal assessment to confirm that the work is his or her authentic work and constitutes the final version of that work. Once a student has officially submitted the final version of the work to a teacher (or the coordinator) for internal assessment, together with the signed coversheet, it cannot be retracted.

Authenticity may be checked by discussion with the student on the content of the work, and scrutiny of one or more of the following:

- the student's initial proposal
- the first draft of the written work
- the references cited
- the style of writing compared with work known to be that of the student.

The requirement for teachers and students to sign the coversheet for internal assessment applies to the work of all students, not just the sample work that will be submitted to an examiner for the purpose of moderation. If the teacher and student sign a coversheet, but there is a comment to the effect that the work may not be authentic, the student will not be eligible for a mark in that component and no grade will be awarded.

The same piece of work cannot be submitted to meet the requirements of both the internal assessment and the extended essay.

Group work

Group work should not be used for explorations. Each exploration is an individual piece of work.It should be made clear to students that all work connected with the exploration, including the writing of the exploration, should be their own. It is therefore helpful if teachers try to encourage in students a sense of responsibility for their own learning so that they accept a degree of ownership and take pride in their own work.

Time allocation

Internal assessment is an integral part of the mathematicsHL course, contributing 20% to the final assessment in the course. This weighting should be reflected in the time that is allocated to teaching the knowledge, skills and understanding required to undertake the work as well as the total time allocated to carry out the work. It is recommended that a total of approximately 10-15 hours of teaching time should be allocated to the work This should include:

- time for the teacher to explain to students the requirements of the exploration
- class time for students to work on the exploration
- time for consultation between the teacher and each student
- time to review and monitor progress, and to check authenticity.

Using assessment criteria for internal assessment

For internal assessment, a number of assessment criteria have been identified. Each assessment criterion has level descriptors describing specific levels of achievement together with an appropriate range of marks. The level descriptors concentrate on positive achievement, although for the lower levels failure to achieve may be included in the description.

Level descriptors:

- The aim is to find, for each criterion, the descriptor that conveys most accurately the level attained by the student.
- When assessing a student's work, teachers should read the level descriptors for each criterion, starting with level 0, until they reach a descriptor that describes a level of achievement that has not been reached.The level of achievement gained by the student is therefore the preceding one, and it is this that should be recorded.
- Only whole numbers should be recorded; partial marks, that is fractions and decimals, are not acceptable.
- Teachers should not think in terms of a pass or fail boundary, but should concentrate on identifying the appropriate descriptor for each assessment criterion.
- The highest level descriptors do not imply faultless performance but should be achievable by a student.
- Teachers should not hesitate to use the extremes if they are appropriate descriptions of the work being assessed.
- A student who attains a high level of achievement in relation to one criterion will not necessarily attain high levels of achievement in relation to the other criteria. Similarly, a student who attains a low level of achievement for one criterion will not necessarily attain low achievement levels for the other criteria.
  Teachers should not assume that the overall assessment of the students will produce any particular distribution of marks.
- It is expected that the assessment criteria be made available to students.
  Internal assessment details

Mathematical exploration

Duration: 10 to 15 teaching hours
Weighting: 20%

The internally assessed component in this course is a mathematical exploration. This is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication (including formulae, diagrams, graphs and so on), with accompanying commentary, good mathematical writing and thoughtful reflection. A student should develop his or her own focus, with the teacher providing feedback via, for example, discussion and interview. This will allow the students to develop area(s) of interest to them without a time constraint as in an examination, and allow all students to experience a feeling of success.

The final report should be approximately 12 to 20 pages long. It can be either word processed or handwritten. Students should be able to explain all stages of their work in such a way that demonstrates clear understanding. While there is no requirement that students present their work in class, it should be written in such a way that their peers would be able to follow it fairly easily. The report should include a detailed bibliography, and sources need to be referenced in line with the IB academic honesty policy. Direct quotes must be acknowledged.

The aim of the exploration

The aims of the Mathematics: analysis and approaches and Mathematics: applications and interpretation courses at both SL and HL are carried through into the objectives that are formally assessed as part of the course, through either written examination papers or the exploration, or both. In addition to testing the objectives of the course, the exploration is intended to provide students with opportunities to increase their understanding of mathematical concepts and processes, and to develop a wider appreciation of mathematics. These are noted in the aims of the course. It is intended that, by doing the exploration, students benefit from the mathematical activities undertaken and find them both stimulating and rewarding. It will enable students to acquire the attributes of the IB learner profile. The specific purposes of the exploration are to:

- develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
- provide opportunities for students to complete a piece of mathematical work over an extended period of time
- enable students to experience the satisfaction of applying mathematical processes independently
- provide students with the opportunity to experience for themselves the beauty, power and usefulness of mathematics
- encourage students, where appropriate, to discover, use and appreciate the power of technology as a mathematical tool
- enable students to develop the qualities of patience and persistence, and to reflect on the significance of their work
- provide opportunities for students to show, with confidence, how they have developed mathematically.

Management of the exploration

Work for the exploration will be incorporated into the course so that students are given the opportunity to learn the skills needed. Time in class can therefore be used for general discussion of areas of study, as well as familiarizing students with the criteria.

Requirements and recommendations

Students can choose from a wide variety of activities, for example, modelling, investigations and applications of mathematics. To assist students in the choice of a topic, a list of stimuli will be provided. However, students are not restricted to this list.

The exploration should not normally exceed 20 pages, including diagrams and graphs, but excluding the bibliography. However, it is the quality of the mathematical writing that is important, not the length. The teacher will give appropriate guidance at all stages of the exploration by, for example, directing students into more productive routes of inquiry, making suggestions for suitable sources of information, and providing advice on the content and clarity of the exploration in the writing-up stage.

Students’ errors will be guided but not explicitly corrected. Students are expected to consult the teacher throughout the process.

All students should be familiar with the requirements of the exploration and the criteria by which it is assessed.Students need to start planning their explorations as early as possible in the course and must stick with the deadlines for the date for he submission of the exploration topic and a brief outline description, the for the submission of the first draft and, the date for completion.

In developing their explorations, students should aim to make use of mathematics learned as part of the course. The mathematics used should be the level of the course and similar to that suggested by the syllabus. It is not expected that students produce work that is outside the mathematics HL syllabus—however, this is not penalized.

Internal assessment criteria

The exploration is internally assessed by the teacher and externally moderated by the IB using assessment criteria that relate to the objectives for mathematics HL.

Each exploration is assessed against the following five criteria. The final mark for each exploration is the sum of the scores for each criterion. The maximum possible final mark is 20. Students will not receive a grade for mathematics HL if they have not submitted an exploration.

Criterion A	Presentation
Criterion B	Mathematical communication
Criterion C	Personal engagement
Criterion D	Reflection
Criterion E	Use of mathematics

Criterion A: Presentation

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence or some organization.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, and concise.

The “presentation” criterion assesses the organization and coherence of the exploration. A coherent exploration is logically developed, easy to follow and meets its aim. This refers to the overall structure or framework, including introduction, body, conclusion and how well the different parts link to each other. A well-organized exploration includes an introduction, describes the aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. Appendices should be used to include information on large data sets, additional graphs, diagrams and tables.

A concise exploration does not show irrelevant or unnecessary repetitive calculations, graphs or descriptions. The use of technology is not required but encouraged where appropriate. However, the use of analytic approaches rather than technological ones does not necessarily mean lack of conciseness, and should not be penalized. This does not mean that repetitive calculations are condoned.

Criterion B: Mathematical communication

The “mathematical communication” criterion assesses to what extent the student has:

- used appropriate mathematical language (notation, symbols, terminology). Calculator and computer notation is acceptable only if it is software generated. Otherwise it is expected that students use appropriate mathematical notation in their work
- defined key terms and variables, where required
- used multiple forms of mathematical representation, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
- used a deductive method and set out proofs logically where appropriate

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration contains some relevant mathematical communication which is partially appropriate.
2	The exploration contains some relevant appropriate mathematical communication.
3	The mathematical communication is relevant, appropriate and is mostly consistent.
4	The mathematical communication is relevant, appropriate and consistent throughout.

Criterion C: Personal engagement

The “personal engagement” criterion assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort. Personal engagement may be recognized in different ways.

There must be evidence of personal engagement demonstrated in the student’s work. It is not sufficient that a teacher comments that a student was highly engaged.

Textbook style explorations or reproduction of readily available mathematics without the candidate’s own perspective are unlikely to achieve the higher levels.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of some personal engagement.
2	There is evidence of significant personal engagement.
3	There is evidence of outstanding personal engagement.

Criterion D: Reflection

This criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited or superficial reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection

Criterion E: Use of mathematics

The "Use of mathematics" HL criterion assesses to what extent students use relevant mathematics in the exploration.

Students are expected to produce work that is commensurate with the level of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, at a similar level or slightly beyond. However, mathematics of a level slightly beyond the syllabus is not required to achieve the highest levels.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used. Limited understanding is demonstrated.
2	Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Some knowledge and understanding are demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and demonstrates sophistication or rigour. Thorough knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and demonstrates sophistication and rigour. Thorough knowledge and understanding are demonstrated.

Mathematics and theory of knowledge

The Theory of knowledge guide identifies four ways of knowing, and it could be claimed that these all have some role in the acquisition of mathematical knowledge. While perhaps initially inspired by data from sense perception, mathematics is dominated by reason, and some mathematicians argue that their subject is a language, that it is, in some sense, universal. However, there is also no doubt that mathematicians perceive beauty in mathematics, and that emotion can be a strong driver in the search for mathematical knowledge. As an area of knowledge, mathematics seems to supply a certainty perhaps missing in other disciplines. This may be related to the “purity” of the subject that makes it sometimes seem divorced from reality. However, Mathematics has also provided important knowledge about the world, and the use of mathematics in science and technology has been one of the driving forces for scientific advances. Despite all its undoubted power for understanding and change, mathematics is in the end a puzzling phenomenon. A fundamental question for all knowers is whether mathematical knowledge really exists independently of our thinking about it. Is it there “waiting to be discovered” or is it a human creation? Students’ attention should be drawn to questions relating theory of knowledge (TOK) and mathematics, and they should be encouraged to raise such questions themselves, in mathematics and TOK classes. This includes questioning all the claims made above.

Mathematics and the international mindedness

Mathematics is in a sense an international language, and, apart from slightly differing notation, mathematicians from around the world can communicate within their field. Mathematics transcends politics, religion and nationality, yet throughout history great civilizations owe their success in part to their mathematicians being able to create and maintain complex social and architectural structures. Despite recent advances in the development of information and communication technologies, the global exchange of mathematical information and ideas is not a new phenomenon and has been essential to the progress of mathematics. Indeed, many of the foundations of modern mathematics were laid many centuries ago by Arabic, Greek, Indian and Chinese civilizations, among others. Teachers could use timeline websites to show the contributions of different civilizations to mathematics, but not just for their mathematical content. Illustrating the characters and personalities of the mathematicians concerned and the historical context in which they worked brings home the human and cultural dimension of mathematics.

The importance of science and technology in the everyday world is clear, but the vital role of mathematics is not so well recognized. It is the language of science, and underpins most developments in science and technology. A good example of this is the digital revolution, which is transforming the world, as it is all based on the binary number system in mathematics. Many international bodies now exist to promote mathematics. Students are encouraged to access the extensive websites of international mathematical organizations to enhance their appreciation of the international dimension and to engage in the global issues surrounding the subject.

Glossary of command terms

Calculate: Obtain a numerical answer showing the relevant stages in the working.

Comment: Give a judgment based on a given statement or result of a calculation.

Compare: Give an account of the similarities between two (or more) items or situations, referring to both (all) of them throughout.

Compare and contrast: Give an account of the similarities and differences between two (or more) items or situations, referring to both (all) of them throughout.

Construct Display information in a diagrammatic or logical form.

Contrast Give an account of the differences between two (or more) items or situations, referring to both (all) of them throughout.

Deduce Reach a conclusion from the information given.

Demonstrate Make clear by reasoning or evidence, illustrating with examples or practical application.

Describe Give a detailed account.

Determine Obtain the only possible answer.

Differentiate Obtain the derivative of a function.

Distinguish Make clear the differences between two or more concepts or items.

Draw Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.

Estimate Obtain an approximate value.

Explain Give a detailed account, including reasons or causes.

Find Obtain an answer, showing relevant stages in the working.

Hence Use the preceding work to obtain the required result.

Hence or otherwise It is suggested that the preceding work is used, but other methods could also receive credit.

Identify Provide an answer from a number of possibilities.

Integrate Obtain the integral of a function.

Interpret Use knowledge and understanding to recognize trends and draw conclusions from given information.

Investigate Observe, study, or make a detailed and systematic examination, in order to establish facts and reach new conclusions.

Justify Give valid reasons or evidence to support an answer or conclusion.

Label Add labels to a diagram.

List Give a sequence of brief answers with no explanation.

Plot Mark the position of points on a diagram.

Predict Give an expected result.

Show Give the steps in a calculation or derivation.

Show that Obtain the required result (possibly using information given) without the formality

of proof. “Show that” questions do not generally require the use of a calculator.

Sketch Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.

Solve Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

State Give a specific name, value or other brief answer without explanation or calculation.

Suggest Propose a solution, hypothesis or other possible answer.

Verify Provide evidence that validates the result.

Write down Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

Notation list

Of the various notations in use, the IB has chosen to adopt a system of notation based on the recommendations of the International Organization for Standardization (ISO). This notation is used in the examination papers for this course without explanation. If forms of notation other than those listedin this course booklet are used on a particular examination paper, they are defined within the question in which

they appear. Studentsare **not** allowed access to information about this notation in the examinations. Students must always use correct mathematical notation, not calculator notation.

N	the set of positive integers and zero, {0,1, 2, 3, ...}
Z	the set of integers, {0, ±1, ± 2, ± 3, ...}
Z^+	the set of positive integers, {1, 2, 3, ...}
Q	the set of rational numbers
Q^+	the set of positive rational numbers, $\{x \mid x \in \mathrm{Q}, x > 0\}$
R	the set of real numbers
R^+	the set of positive real numbers, $\{x \mid x \in \mathrm{R}, x > 0\}$
$\{x_1, x_2 ...\}$	the set with elements x_1, x_2 ...
n(*A*)	the number of elements in the finite set *A*
{*x* | }	the set of all *x* such that
∈	is an element of

$\notin$	is not an element of
$\emptyset$	the empty (null) set
U	the universal set
$\cup$	Union
$\cap$	Intersection
$\subset$	is a proper subset of
$\subseteq$	is a subset of
A'	the complement of the set *A*
a | b	a divides b
$a^{\frac{1}{n}}$, $\sqrt[n]{a}$	*a* to the power of $\frac{1}{n}$, *n* th root of *a* (if $a \geq 0$ then $\sqrt[n]{a} \geq 0$)
$\|x\|$	modulus or absolute value of *x* , that is x for $x \geq 0, x \in \mathbb{R}$, and -x for $x < 0$, $x \in \mathbb{R}$,
$\approx$	is approximately equal to
$>$	is greater than
$\geq$	is greater than or equal to
$<$	is less than
$\leq$	is less than or equal to
$\not>$	is not greater than
$\not<$	is not less than
U_n	the n^{th} term of a sequence or series
d	the common difference of an arithmetic sequence
r	the common ratio of a geometric sequence
S_n	the sum of the first *n* terms of a sequence, $u_1 + u_2 + \ldots + u_n$
S_∞	the sum to infinity of a sequence, $u_1 + u_2 + \ldots$
$\sum_{i=1}^{n} u_i$	$u_1 + u_2 + \ldots + u_n$
$\binom{n}{r}$	the *r* th binomial coefficient, *r* = 0, 1, 2, …, in the expansion of $(a + b)^n$
$f : A \rightarrow B$	*f* is a function under which each element of set *A* has an image in set *B*
$f : x \mapsto y$	*f* is a function under which *x* is mapped to *y*
$f(x)$	the image of *x* under the function *f*
f^{-1}	the inverse function of the function *f*
$f \circ g$	the composite function of *f* and *g*
$\lim_{x \to a} f(x)$	the limit of *f (x) as x tends to a*
$\frac{dy}{dx}$	the derivative of y with respect to x
$f'(x)$	the derivative of *f* (*x*) with respect to *x*
$\frac{d^2y}{dx^2}$	the second derivative of *y* with respect to *x*
$f''(x)$	the second derivative of *f* (*x*) with respect to *x*
$\frac{d^ny}{dx^n}$	the n^{th} derivative of *f* (*x*) with respect to *x*
$f^n(x)$	the n^{th} derivative of *f* (*x*) with respect to *x*

$\int y\, dx$	the indefinite integral of *y* with respect to *x*
$\int_a^b y.dx$	the definite integral of *y* with respect to *x* between the limits *x* = *a* and *x* = *b*
e^x	exponential function (base e) of *x*
$\log_a x$	logarithm to the base *a* of *x*
ln *x*	the natural logarithm of *x* , $\log_e x$
sin, cos, tan	the circular functions
A(*x*, *y*)	the point A in the plane with Cartesian coordinates *x* and *y*
[AB]	the line segment with end points A and B
|AB|	the length of [AB]
(AB)	the line containing points A and B
Â	the angle at A
$\widehat{CAB}$	the angle between [CA] and [AB]
ΔABC	the triangle whose vertices are A , B and C
$\vec{\boldsymbol{v}}$	the vector $\vec{\boldsymbol{v}}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment from A to B
$\vec{\boldsymbol{a}}$	the position vector $\overrightarrow{OA}$
$\vec{\boldsymbol{i}}, \vec{\boldsymbol{j}}, \vec{\boldsymbol{k}}$	unit vectors in the directions of the Cartesian coordinate axes
$\|\vec{\boldsymbol{a}}\|$	the magnitude of **a**
$\overrightarrow{\|AB\|}$	the magnitude of AB
$\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{w}}$	the scalar product of ***v*** and ***w***
P(*A*)	probability of event *A*
P(*A′*)	probability of the event “not *A* ”
P(*A*| *B*)	probability of the event *A* given the event *B*
$x_1, x_2, \ldots$	Observations
$f_1, f_2, \ldots$	frequencies with which the observations, $x_1, x_2,$ ... occur
$\binom{n}{r}$	number of ways of selecting r items from n items
B(*n*, *p*)	binomial distribution with parameters *n* and *p*
N(μ,σ 2)	normal distribution with mean μ and variance σ 2
X ~ B(*n*, *p*)	the random variable *X* has a binomial distribution with parameters *n* and *p*
X ~ N(μ ,σ 2)	the random variable *X* has a normal distribution with mean μ and variance σ 2
μ	population mean
σ^2	population variance
σ	population standard deviation
x	mean of a set of data, $x_1, x_2, x_3,$............
z	standardized normal random variable, z= $\frac{x\ \mu}{\sigma}$
Φ	cumulative distribution function of the standardized normal variable with distribution N(0, 1)
r	Pearson’s product–moment correlation coefficient

Materials

Use of calculators (TI-84 PLUS SILVER EDITION or TI-84 PLUS CE-T)

Students are expected to have access to a graphic display calculator (GDC) at all times during the course.

Mathematics HL formula booklet
Each student is required to have access to a clean copy of this booklet during the examination. Students should be familiar with the contents of this document from the beginning of the course.

Command terms and notation list

Students need to be familiar with the IB notation and the command terms, as these will be used without explanation in the examination papers.

Resources
Library resources relating to mathematics are listed below:

Dmrbs No.	Title	Author
05A20269	Sciencia : mathematics, physics, chemistry, biology, and	Burkard Polster
05A19762	Reading the book of nature : an introduction to the	Peter Kosso
05A20321	1089 and all that : a journey into mathematics	David J. Acheson
05A20307	Alex’s adventures in numberland	Alex Bellos
05A20254	Mathematical studies : standard level : course	Peter Blythe
05A19454	Calculus	William L. Briggs
05A19459	Calculus	William L. Briggs
05A19749	Mathematics standard level : course companion : IB	Laurie Buchanan
05A20342	Mathematics standard level : course companion : IB	Laurie Buchanan
05A19433	Prealgebra	Tom Carson
05A20305	50 mathematical ideas : you really need to know	Tony Crilly
05A20508	The mathematical experience	Philip J. Davis
05A20507	The mathematical experience	Philip J. Davis
05A19478	Schaum's outline of basic business mathematics	Eugene Don
05A19755	Mathematics standard level for the IB diploma :	Paul Fannon
05A20325	Mathematics : a very short introduction	Timothy Gowers
05A20294	Additional mathematics for OCR	Val Hanrahan
05A20344	Mathematics higher level : course companion : IB	Josip Harcet
05A19473	University calculus: elements with early transcendentals	Joel Hass
05A20395	God created the integers : the mathematical	Stephen W. Hawking
05A20322	Nets, puzzles, and postmen : an exploration of	Peter M. Higgins
05A20004	Where mathematics come from : how the embodied	George Lakoff
05A19490	Finite math with applications	Margaret L. Lial

05A19469	Schaum's outline of discrete mathematics	Seymour Lipschutz
05A19495	Maths quest 12: mathematical methods	Jennifer Nolan
05A20338	Mathematics for the international student : mathematics	John Owen
05A20271	The Princeton companion to mathematics	
05A20298	The number mysteries : a mathematical Odyssey	Marcus Du Sautoy
05A20267	Mathematical miniatures	Svetoslav Savchev
05A19476	Schaum's outline of college mathematics	Philip Schmidt
05A19483	Schaum's outline of mathematical handbook of formulas	Murray Spiegel
05A20316	Professor Stewart's hoard of mathematical treasures	Ian Stewart
05A20309	The magical maze : seeing the world through	Ian Stewart
05A20323	How to cut a cake : and other mathematical conundrums	Ian Stewart
05A20397	Professor Stewart's cabinet of mathematical curiosities	Ian Stewart
05A19500	Essential mathematics for economic analysis	Knut Sydsaeter
05A20363	Mathematics for the international students : IB dipolma	Paul Urban
05A19498	Finite mathematics: an applied approach	Paula Grafton Young
05A21021	The concise Oxford dictionary of mathematics	Christopher Clapham
05A20394	The IMO compendium : a collection of problems	Dusan Djukic
05A20304	Taming the infinite : the story of mathematics from the	Ian Stewart
05A20331	Zero : the biography of a dangerous idea	Charles Seife
05A21013	Proofs and refutations : the logic of mathematical	Imre Lakatos

Teachers Responsible

Mehriban Kılıç mehriban.kilic@fmvisik.k12.tr

Burcu Baydili burcu.baydili@fmvisik.k12.tr